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On Optical Dispersion Formulæ.

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In the half century that elapsed from the time when Cauchy published his memoirs on dispersion, the only measurements of refractive indices that were available for the test of any theory were confined to the neighbourhood of the visible spectrum. They extended from somewhat beyond 0.2μ to about 0.8μ , *i.e.* over a range of two octaves. In this range the variation of refractive index for most substances is small, and it is not surprising that, by a proper adjustment of constants, a variety of different formulæ may be made to fit the experimental facts. More recently, however, the range of observations has been immensely extended, so that we now have a large number of determinations of refractive indices for various wave-lengths extending from less than 0.2μ to 22.3μ . With a knowledge extending over this range of about seven octaves, there is more hope of testing the merits of different dispersion formulæ. Accordingly, when, in 1879, Mouton made the first considerable extension in the direction of great wave-lengths, he found that, with the same number of constants, the formula of Briot represented the facts much better than did the older one of Cauchy, and the same result was reached by Langley in 1884 and 1886. The still more extended observations of Paschen, Rubens, E. F. Nichols, and A. Trowbridge, published at various dates from 1892 to 1897, have shown that Sellmeier's formula*

$$n^2 = K + c_1/(\lambda^2 - \lambda_1^2) + \dots$$

is the only one of those that have been tried that is in thorough accord with the observed values of the refractive indices over the whole range of the experiments.

It must, however, be admitted that there are various doubts and difficulties connected with the development of this formula from a sound dynamical basis in any of the various forms of its presentation, whether we follow directly the arguments of Sellmeier, Ketteler, Helmholtz, or Kelvin, or express their ideas in the language of electron theory as Drude and others have done. But apart from difficulties as to the exact character of the reaction between ether and matter, and doubts as to the validity of certain

* Usually quoted now as the "Ketteler-Helmholtz formula."

dynamical principles employed in its evolution, the Sellmeier formula is open to two serious objections.

(1) When the wave-length is indefinitely increased, the formula gives $n^2 = K$, so that, on the electromagnetic theory of light, K should be the dielectric constant of the medium. The values of K , determined by the Sellmeier formula from the observations of refractive indices by Rubens and Nichols, are 6.09, 4.58, 5.18, and 4.55 for fluorite, quartz, rocksalt, and sylvin respectively, and the slightly different observations of the other physicists mentioned above make scarcely any appreciable change in the estimate of K . Thus Langley finds $K = 5.1747$ for rocksalt, while Rubens and Trowbridge find $K = 5.179$. Unfortunately there is not the same agreement amongst different direct determinations of the dielectric constant. For fluorite the constant is 6.8, 6.7, or 6.9, according to Curie, Romich and Nowak, or Starke; while for quartz these observers find 4.55, 4.6, and 4.73 respectively. For rocksalt Curie gets 5.85, Thwing 5.81, and Starke 6.29; and the last named observer gives 4.94 as the dielectric constant of sylvin. Taking the mean of these for the different substances, we find 6.8, 4.63, 5.98, and 4.94, as compared with 6.09, 4.58, 5.18, and 4.55 of the dispersion formula. It will be noted that the differences are in all cases in the same direction, and, except in the case of quartz, are considerable, being more than 10 per cent. of the quantity considered.

(2) A more serious discrepancy reveals itself on comparing the calculated values of λ_2 and the observed positions of the absorption bands of different substances. According to the theory, the absorption band should be found very near to λ_2 , although not absolutely coincident with it. The values of λ_2 , calculated from the formula to fit in with the experimental determinations of refractive indices by Nichols, Rubens, and Paschen for rocksalt, sylvin, and fluorite are 56.12, 67.21, and 35.47 respectively, whereas the absorption bands were found at 51.2, 61.1, and 23.7. Here, again, the differences are in all cases in the same direction, and are of an order of magnitude altogether beyond the limits of experimental error. The difference is greatest for fluorite, where it amounts to nearly 12μ , but the smallest difference, about 5μ for rocksalt, is at least five times too great to be ascribed to errors in the experiments. It must be borne in mind that the formula involves not λ_2 but its square, so that the observed and computed values that are to be compared are 2621, 3733, and 562, with 3149, 4517, and 1258 respectively for the three substances mentioned. The substitution of the former for the latter numbers in the formula will make a very great change indeed, and throw the observed and computed values of the refractive indices into hopeless disagreement.

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If we are to retain the Sellmeier formula, only two methods of escape from these difficulties seem possible. One is to ascribe the discrepancies to experimental errors. This is, however, quite out of the question as regards the position of the absorption bands, and there is still less hope of reconciliation from the very slight differences in the measurements of the refractive indices. The greatest differences found in the refractive indices of rocksalt are those between the measurements of Langley, on the one hand, and Rubens, Nichols and Trowbridge on the other. The consequent changes in the estimates of K and λ_2^2 from the Sellmeier formula, are, however, very slight, for, while Rubens gets $K=5.179$ and $\lambda_2^2=3149.3$, Langley has $K=5.175$ and $\lambda_2^2=3145.7$.

The other mode of escape that has been suggested is to ascribe the discrepancy between the observed and calculated values of λ_2 to the influence of absorption bands in the unexplored regions. The Sellmeier formula contains a term $c_n/(\lambda^2-\lambda_n^2)$, corresponding to each absorption band, and one or more of these terms may be neglected in estimating the refractive index in any region, on the supposition that $\lambda^2-\lambda_n^2$ is very large, which means, of course, that the region is far removed from the absorption bands in question. However, there is no evidence of the existence of any absorption band in a position that would enable us to bridge the gulf between theory and experiment for either rocksalt or sylvine.* In the case of fluorite, there is a well-marked band in the neighbourhood of $24\ \mu$ and strong absorption for a considerable distance beyond, but not reaching nearly so far as $35.47\ \mu$. The only one of the substances mentioned for which an absorption band has been found in a region that should affect the calculated refractive index is quartz. This has an absorption band at $12.5\ \mu$, which was not considered in the calculations quoted above. Quartz, however, is the substance in which the agreement between Sellmeier's formula and the observed values of the dielectric constant and the position of the other absorption bands is most satisfactory.

* Quite recently Paschen has made another interesting contribution to the discussion of dispersion formulæ. In the 'Annalen der Physik' (1908, p. 120) he deals with rocksalt and sylvine on the basis of Sellmeier's formula. By assuming that there are three absorption bands—one in the infra-red and two in the ultra-violet—he has seven constants at his disposal, and obtains a close agreement between theory and observation as far as the dispersion is concerned. However, the difficulties as to the values of the constants in the formula do not seem to be removed. The dielectric constant is still too low, viz., 3.87, instead of 4.94, for sylvine, and 5.68, instead of 5.98, for rocksalt (taking the mean of the observations quoted above). The absorption bands of rocksalt are placed at $0.12\ \mu$, $0.16\ \mu$, and $60\ \mu$, and those of sylvine at $0.11\ \mu$, $0.16\ \mu$, and $57\ \mu$. Lyman has experimented with wave-lengths much less than $0.16\ \mu$ without finding any absorption bands, and the most accurate observers of the bands in the infra-red agree in placing them a long way from $60\ \mu$ and $57\ \mu$.

It seems then worth while inquiring whether there is not some other dispersion formula that will agree satisfactorily with the observed values of the refractive indices and be free from the difficulties in which Sellmeier's formula is involved. In order to make clear the meaning of the symbols, it may be well to sketch the process by which the formula that we propose to test may be derived.

Let ξ, η, ζ denote the components of the displacement in an ether that resists rotations only. If, for convenience, we take the density of this ether to be unity, the torque F necessary to produce a rotation f is $F = c^2 f$ and the work required to do this is $\frac{1}{2} c^2 f^2$. Here c^2 is a coefficient of elasticity and c is the velocity of a wave in the ether. If there be electrons present, these will be disturbed so as to produce and equilibrate a new averaged rotational strain f' . The total rotation in the elastic ether is thus $f + f'$, say f_1 , while the torque is $F_1 = c^2 f$. The quantity f' will, on the usual hypothesis of very small displacements, be proportional to f , so that we may write $f' = (n^2 - 1)f$, where n is some constant. We thus have $F_1 = \frac{c^2}{n^2} f_1$, where $f_1 = n^2 f$. The work absorbed in producing the total rotation f_1 , for the compound medium made up of ether and electrons, will be $\frac{1}{2} \frac{c^2}{n^2} f_1^2$. Thus the c of free ether is replaced by c/n for the compound medium and n represents the refractive index. If we employ the language of electrodynamics, F is to be identified with the electromotive force and the velocity $(\xi \dot{\eta} \zeta)$ with the magnetic induction. In estimating the actual local electromotive force effective in straining any molecule P it is convenient, as in the theory of attractions, to consider separately the influence of the electrons within the region adjacent to P and that of the external field beyond this. What exactly the contribution of adjacent molecules is will depend on the character of the intermolecular forces and the configuration around each molecule; but in any case it must be proportional to f' , and it will be compensated locally by static reaction on the adjacent molecules. Hence the actual electromotive force around the molecule is

$$F' = c^2 (f + f' - a_1 f') = m f' (n^2 + a) / (n^2 - 1),$$

where $m = (1 - a_1) c^2$ and $a = a_1 / (1 - a_1)$. We thus have

$$(n^2 - 1) / (n^2 + a) = m f' / F',$$

so that the refractive index (n) depends on the relation between f' and the torque F' that produces it.

To find the relation between f' and F' we have to consider the effect on the molecule of the alternating field of force due to the impinging wave of light.

This is the well-known problem of determining the oscillations forced on a vibrating system in steady motion by a given periodic force. The principle of least action lends itself most readily to such a discussion and enables us to arrive at the form of the relation sought for without any special hypothesis as to the character of the steady motion of the electrons, or as to the nature of the intermolecular forces. If $p/2\pi$ be the frequency of the impinging wave and λ the corresponding wave-length, we are thus led to the formula*

$$\begin{aligned}\frac{n^2-1}{n^2+a} &= \frac{mf'}{F'} = \frac{A_1}{p^2-p_1^2} + \frac{A_2}{p^2-p_2^2} + \dots + \frac{A_n}{p^2-p_n^2} \\ &= \frac{B_1\lambda^2}{\lambda^2-\lambda_1^2} + \frac{B_2\lambda^2}{\lambda^2-\lambda_2^2} + \dots + \frac{B_n\lambda^2}{\lambda^2-\lambda_n^2},\end{aligned}$$

where $p_1/2\pi, \dots, p_n/2\pi$ are the frequencies of the free vibrations within the molecule. For indefinitely long waves we have $n^2 = K$, so that

$$\frac{K-1}{K+a} = B_1 + B_2 + \dots + B_n,$$

whence, by subtraction,

$$\frac{n^2-1}{n^2+a} = \frac{K-1}{K+a} + \frac{c_1}{\lambda^2-\lambda_1^2} + \frac{c_2}{\lambda^2-\lambda_2^2} + \dots + \frac{c_n}{\lambda^2-\lambda_n^2}.$$

This formula should give the refractive indices for all the different wave-lengths for which the medium is transparent. The formula will be modified in the same way as that of Sellmeier in the neighbourhood of an absorption band. If anything occurs to put f' out of phase with f , then $p_1, \dots$ will be complex and there will be general absorption of the metallic type.

It has been remarked that the value of the constant a depends on the influence of the electrons in the immediate vicinity of the point where the disturbance is considered. In the present state of our ignorance as to the arrangement and mutual influence of the molecules the value of this constant cannot be accurately determined from theory. In the case of a liquid or gas we might expect the average effect to be that of a uniform distribution and under these circumstances Larmor† has given reasons for expecting the constant a to have the value 2. If $a = 2$ we have the expression $(n^2-1)/(n^2+2)$, which is what occurs in Lorentz's original paper‡ connecting refractive index (and dispersion) with density. The experiments there discussed, and the more recent estimates of the refractive indices of liquids and gases agree, on the whole, fairly closely with Lorentz's

* Cf. Larmor, 'Phil. Trans.,' 1897, A, p. 236, for an investigation in detail. What is here denoted by a is Larmor's $4\pi/\lambda - 1$.

† *Loc. cit.*, p. 233.

‡ H. A. Lorentz, 'Ann. der Physik u. Chemie,' 1879, vol. 9, p. 641.

theoretical formula. With these considerations before us it would seem natural to inquire whether the formula

$$\frac{n^2-1}{n^2+2} = \frac{K-1}{K+2} + \frac{c_1}{\lambda^2-\lambda_1^2} + \dots + \frac{c_n}{\lambda^2-\lambda_n^2}$$

will apply to solids as well as liquids. However, on attempting this on the basis of the experiments referred to below, we are met with the same difficulties that confronted us when considering Sellmeier's formula. A fair agreement between theory and observation can be obtained as far as the dispersion is concerned, but the constants are unsatisfactory. Thus in the case of rocksalt, if we take any probable value of the dielectric constant, the formula predicts an absorption band in the neighbourhood of 46μ and this is as much below the mark as that given by Sellmeier's formula is above it. We shall therefore retain the constant a in our formula and endeavour to determine its value for different substances from the experimental evidence that is available.

The substance best adapted for the test of any such formula is rocksalt. Its refractive indices are known to a high degree of accuracy over a very large range, within which there are only two free periods as compared with four in the case of quartz. The addition of a free period increases the number of unknown constants, and makes it more difficult to determine any of them accurately.

Amongst the most careful determinations of the dispersion of rocksalt we have the observations of Langley* from 0.486μ to 4.123μ , of Rubens† from 0.434μ to 5.746μ , of Paschen‡ from 2.8μ to 9.76μ , of Rubens§ from 0.434μ to 8.95μ , of Rubens and Nichols|| from 0.434μ to 22.3μ , and of Rubens and Trowbridge¶ from 9.88μ to 17.87μ . On the whole there is excellent agreement between these various observers, the differences in the estimates of the refractive indices being rarely more than 3 or 4 in the fifth significant figure. Langley gives the indices to seven significant figures, and his results are admirably self-consistent. Unfortunately, however, they are almost uniformly higher than those of the other observers, so that it is useless at present to include more than five figures in calculating the values of n from our formula. The influence of small differences in the fifth significant figure

* 'American Journal of Science,' 1886, vol. 32, p. 98, and 'Annals of Astrophysical Observatory of Smithsonian Institution,' 1900, vol. 1, p. 261.

† 'Ann. der Physik u. Chemie,' 1892, vol. 45, p. 254.

‡ *Ibid.*, 1894, vol. 53, p. 340.

§ *Ibid.*, 1894, vol. 53, p. 278; and 1895, vol. 54, p. 482.

|| *Ibid.*, 1897, vol. 60, p. 454; and 'Physical Review,' 1897, vol. 5, p. 162.

¶ *Ibid.*, 1897, vol. 60, p. 733; and corrections in vol. 61, p. 224.

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on the estimated values of the constants will, however, as has been indicated already, be very slight, and it is almost immaterial which set of observations we use as the basis of our computation.

If we take Langley's figures, our problem is to determine the six constants K , a , λ_1^2 , λ_2^2 , c_1 , and c_2 , from the formula

$$\frac{n^2-1}{n^2+a} = \frac{K-1}{K+a} + \frac{c_1}{\lambda^2-\lambda_1^2} + \frac{c_2}{\lambda^2-\lambda_2^2},$$

so as to get the best possible agreement with over 50 observations of the refractive index (n) for different wave-lengths. To pick out six different observations and solve the corresponding set of six equations for the unknown constants would be possible but very laborious. After the eliminations were performed the resulting equations, of whatever degree, would have numerical coefficients, and so could be solved; but the process of elimination would be very troublesome. In practice it is more feasible to begin by trial and error, and to proceed by successive approximations. It soon becomes evident that the constant c_1 is small, and that for large values of λ the term $c_1/(\lambda^2-\lambda_1^2)$ is almost negligible compared with $c_2/(\lambda^2-\lambda_2^2)$. Thus for the longer wave-lengths we have the simpler formula

$$\frac{n^2-1}{n^2+a} = \frac{K-1}{K+a} + \frac{c_2}{\lambda^2-\lambda_2^2}.$$

Setting out, then, with any arbitrary values of a and K , we can determine the other constants to fit in with the experimental results, and proceed with this process until a set of values is reached that gives a close agreement throughout the whole range. Once such an agreement has been reached, we may reduce the equations for the small differences in the constants to a linear form by the process usual in such problems, and, if required, can determine the probable error of the various estimates.

It might seem at first that with so many constants at our disposal it would be possible to vary their values considerably without producing an appreciable change in the calculated values of the refractive indices. It will be found, however, on trial that the observations are sufficient to confine the constants within narrow ranges, and, amongst other things, that a slight diminution of the constant a would make λ_1^2 negative, which is, of course, impossible. The process of trial and error described above leads to the following values of the constants :—

$$\begin{array}{ll} \lambda_1^2 = 0.0160074, & \lambda_2^2 = 2632.14, \\ c_1 = 0.00191605, & c_2 = 683.816, \\ a = 5.51, & K = 5.9, \\ \lambda_1 = 0.12652, & \lambda_2 = 51.3. \end{array}$$

Calculating the refractive indices from the formula with these constants, we get the following table, which also gives a comparison with Langley's observations. The numbers obtained from theory and observation are identical except in the cases marked with an asterisk:—

λ .	n (theory).	n (observation).	λ .	n (theory).	n (observation).
0·4861 μ	1·5533	1·5533	0·9916 μ	1·5323	1·5323
0·4920	1·5526	1·5526	1·0084	1·5321	1·5321
0·4937	1·5525	1·5525	1·0368	1·5317	1·5317
0·4983	1·5519	1·5519	1·0540	1·5315	1·5315
0·5173	1·5500	1·5500	1·0810	1·5312	1·5312
0·5184	1·5499	1·5499	1·1058	1·5310	1·5310
0·5273	1·5491	1·5491	1·1420	1·53069	1·53063*
0·5372	1·5482	1·5482	1·1780	1·5303	1·5303
0·5530	1·5469	1·5469	1·2016	1·53018	1·53014*
0·5660	1·54586	1·5458*	1·2604	1·5297	1·5297
0·5710	1·5455	1·5455	1·3126	1·5294	1·5294
0·5758	1·5452	1·5452	1·4874	1·5285	1·5285
0·5786	1·5450	1·5450	1·5552	1·52820	1·52814*
0·5860	1·5445	1·5445	1·6368	1·5278	1·5278
0·5893	1·5443	1·5443	1·6848	1·52770	1·52764*
0·6105	1·5430	1·5430	1·7670	1·5274	1·5274
0·6400	1·5414	1·5414	2·0736	1·5265	1·5265
0·6563	1·5406	1·5406	2·1824	1·5263	1·5262*
0·6874	1·5393	1·5393	2·2464	1·5261	1·5261
0·7190	1·53814	1·53815*	2·3560	1·5258	1·5258
0·7604	1·5368	1·5368	3·1104	1·5241	1·5240*
0·7992	1·5358	1·5358	3·2736	1·5237	1·5237
0·8424	1·5348	1·5348	3·3696	1·5235	1·5235
0·8835	1·5340	1·5340	3·6288	1·5229	1·5229
0·9033	1·5336	1·5336	3·8192	1·5224	1·5224
0·9724	1·5325	1·5325	4·1230	1·5216	1·5216

The differences between theory and observation being so slight, and in every case less than those between the different observers, it was not thought necessary to proceed to any closer approximation by modifications of the constants.

We must now inquire how the refractive indices obtained from our formula with these constants agree with the observations of Rubens and others in the region beyond 4·123 μ . The results are set out in the following table:—

λ .	n (theory).	n (observation).	Difference.
4·65 μ	1·5200	1·5197	+ 0·0003
5·22	1·5183	1·5180	+ 0·0003
5·79	1·5155	1·5159	- 0·0004
6·78	1·5123	1·5121	+ 0·0002
7·22	1·5103	1·5102	+ 0·0001
7·59	1·5086	1·5085	+ 0·0001
8·04	1·5063	1·5064	- 0·0001
8·67	1·5030	1·5030	0
9·95	1·4952	1·4951	+ 0·0001
11·88	1·4809	1·4805	+ 0·0004
13·96	1·4625	1·4627	- 0·0002
15·89	1·4415	1·4410	+ 0·0005
17·93	1·4152	1·4148	+ 0·0004
20·57	1·3736	1·3735	+ 0·0001
22·3	1·3407	1·3403	+ 0·0004

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It will be seen that the agreement is very close throughout, the differences being nearly all of the order of experimental error. In considering the slight differences that do exist, we must bear in mind two facts. (1) The formula, with the constants adopted, gives values of n for the larger wave-lengths that are on the average slightly in excess of those observed. This is what is to be expected from the fact that the constants were chosen to fit in with Langley's observations, and in the range common to all observers Langley's results are almost uniformly slightly higher than those of the others. (2) For the larger wave-lengths an error in the estimate of λ affects the refractive index more than in the case of short waves. Thus in the neighbourhood of 18μ an error of one-sixth of 1 per cent. in the estimate of the wave-length would account for a difference of more than 0.0004 in the value of the refractive index. Such a mistake in the estimate of the wave-length is not beyond the limits of experimental error in this region.*

The value of λ_1 adopted to fit in with Langley's observations of the refractive indices is $\lambda_1 = 0.12652\mu$. If we had used the observations of Rubens instead of those of Langley we should have found a slightly lower value of λ_1 . We should thus expect an absorption band in the neighbourhood of 0.12μ . Lyman, in his examination of rocksalt, used wave-lengths as short as 0.125μ . He did not find an absorption band, but he discovered marked absorption for the shorter wave-lengths, indicating that he was on the confines of a band.†

It thus appears that the formula

$$\frac{n^2-1}{n^2+a} = \frac{K-1}{K+a} + \frac{c_1}{\lambda^2-\lambda_1^2} + \frac{c_2}{\lambda^2-\lambda_2^2}$$

is in thorough accord with experimental knowledge on the optical properties of rocksalt. It agrees with experiment in indicating absorption bands in the neighbourhood of 0.12μ , and of 51.3μ ; it suggests a dielectric constant of 5.9, which is very near the mean of the best experimental measurements of that constant; and it assigns values to the indices of refraction agreeing within the limits of experimental error with 70 observations extending over nearly six octaves.

Next to rocksalt, the substance for which the existing data are best adapted for the test of a dispersion formula such as that here discussed is fluorite. Its refractive indices have been carefully determined from 0.198μ

* See Rubens and Nichols, 'Physical Review,' 1897, vol. 5, pp. 102 and 161; and 'Ann. der Phys. u. Chemie,' 1897, vol. 60, pp. 426 and 453.

† 'Astrophysical Journal,' Jan., 1907, pp. 51 and 52.

to 8.95μ by Rubens and Snow,* from 0.1856μ to 9.4291μ by Sarasin, Carvallo and Paschen,† from 0.2μ to 2.34μ by Simon,‡ and from 0.76μ to 3.4μ by Langley.§

Taking the numbers as set out by Paschen as the basis of our calculations, we get the following approximate values of the constants in the dispersion formula:—

$$\begin{aligned} a &= 1.04, & K &= 6.8, \\ \lambda_1^2 &= 0.00716764, & \lambda_2^2 &= 576.353, \\ \lambda_1 &= 0.0846618, & \lambda_2 &= 24.0074, \\ c_1 &= 0.001303, & c_2 &= 231.856. \end{aligned}$$

The corresponding values of the refractive indices for different wavelengths are set out below, with a comparison between theory and observation.

λ .	n (experiment).	n (theory).	Difference.
0.1856	1.5094	1.50945	+0.00005
0.19881	1.4963	1.4962	—0.0001
0.20243	1.4933	1.4932	—0.0001
0.20610	1.4904	1.4904	0
0.20988	1.4876	1.4875	—0.0001
0.21441	1.4846	1.4845	—0.0001
0.22645	1.4776	1.4775	—0.0001
0.23125	1.4752	1.4751	—0.0001
0.25713	1.4648	1.4647	—0.0001
0.27467	1.4596	1.4596	0
0.32525	1.4499	1.4498	—0.0001
0.34015	1.4477	1.4477	0
0.34655	1.4470	1.4469	—0.0001
0.36009	1.4454	1.4454	0
0.39685	1.4421	1.4419	—0.0002
0.48607	1.4371	1.4371	0
0.58932	1.4339	1.4339	0
0.65618	1.4326	1.4327	+0.0001
0.68671	1.4320	1.4320	0
0.71836	1.4316	1.4317	+0.0001
0.76040	1.4310	1.4311	+0.0001
0.8840	1.4298	1.4299	+0.0001
1.1786	1.4279	1.4280	+0.0001
1.3756	1.4269	1.4270	+0.0001
1.4733	1.4264	1.4265	+0.0001
1.5715	1.4260	1.4261	+0.0001
1.7680	1.4251	1.4251	0
1.9153	1.4244	1.4244	0
1.9644	1.4241	1.4241	0
2.0626	1.4236	1.4237	+0.0001
2.1608	1.4231	1.4231	0
2.3573	1.4220	1.4220	0
2.5537	1.4209	1.4209	0
2.6519	1.4202	1.4202	0

* 'Ann. der Phys. u. Chem.,' 1894, vol. 53, p. 273; 1892, vol. 45, p. 254; 1892, vol. 46, p. 529.

† *Ibid.*, 1894, vol. 53, p. 328; 1895, vol. 56, pp. 765 and 821.

‡ *Ibid.*, vol. 53, p. 553.

§ 'Annals of Astr. Obs.,' vol. 1, p. 221; see also Martens, 'Ann. der Phys.,' vol. 6, 1901, p. 619.

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λ .	n (experiment).	n (theory).	Difference.
2·9466	1·4183	1·4183	0
3·2413	1·4161	1·4161	0
3·5359	1·4138	1·4138	0
3·8306	1·4112	1·4112	0
4·1252	1·4085	1·4085	0
4·7146	1·4024	1·4023	-0·0001
5·3036	1·3953	1·3952	-0·0001
5·8932	1·3872	1·3871	-0·0001
6·4825	1·3782	1·3781	-0·0001
7·0718	1·3680	1·3680	0
7·6612	1·3568	1·3568	0
8·2505	1·3444	1·3444	0
8·8398	1·3308	1·3308	0
9·4291	1·3161	1·3161	0

It will be seen that the agreement is very close throughout. It could probably be improved by a slight modification of the constants in the dispersion formula; but it is needless to do this until there is a closer agreement between different observers as to the values of the refractive indices.

The value assigned to the dielectric constant is the mean of the best direct determinations of that quantity, and the prediction as to the positions of the absorption bands is as near the mark as could be desired.
